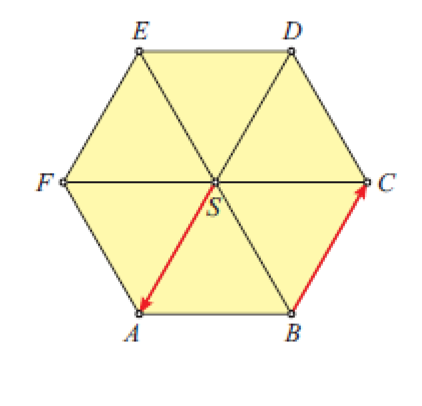
VEKTORI
Sistematizacija gradiva

1. Točka S sjecište je dijagonala pravilnog šesterokuta $ABCDEF$. Izračunaj:



a) $\overrightarrow{AB} + \overrightarrow{AS} + \overrightarrow{AF}$

b) $\overrightarrow{SB} + \overrightarrow{SD} + \overrightarrow{SF}$.

c) $\overrightarrow{AB} - \overrightarrow{DC}$

d) $\overrightarrow{CD} - \overrightarrow{FE}$

Rješenja:

a) $\overrightarrow{AB} + \overrightarrow{AS} + \overrightarrow{AF} = (\overrightarrow{AB} + \overrightarrow{AF}) + \overrightarrow{AS} = \overrightarrow{AS} + \overrightarrow{AS} = 2\overrightarrow{AS} = \overrightarrow{AD}$

b) $\overrightarrow{SB} + \overrightarrow{SD} + \overrightarrow{SF} = (\overrightarrow{SB} + \overrightarrow{SF}) + \overrightarrow{SD} = \overrightarrow{SA} + \overrightarrow{SD} = \overrightarrow{SA} + (-\overrightarrow{SA}) = \vec{0}$

c) $\overrightarrow{AB} - \overrightarrow{DC} = \overrightarrow{AB} - \overrightarrow{SB} = \overrightarrow{AB} + \overrightarrow{BS} = \overrightarrow{AS} = \overrightarrow{BC}$;

d) $\overrightarrow{CD} - \overrightarrow{FE} = \overrightarrow{CD} - \overrightarrow{SD} = \overrightarrow{CD} + \overrightarrow{DS} = \overrightarrow{CS} = \overrightarrow{BA}$;

2.

Točke $A(-1,-1), B(3,-2), C(5,2)$ tri su uzastopna vrha paralelograma ABCD.

a) Kolika je duljina dijagonale $\overline{BD}$?
b) Odredi šiljasti kut paralelograma.

a)

$A(-1,-1),$
$B(3,-2),$
$C(5,2),$
$D(x_D, y_D),$

$|\overline{BD}| = ?$

$\overrightarrow{AB} = \overrightarrow{DC}$
$(3+1)\vec{i} + (-2+1)\vec{j} = (5-x_D)\vec{i} + (2-y_D)\vec{j}$
$4\vec{i} - \vec{j} = (5-x_D)\vec{i} + (2-y_D)\vec{j}$
$5 - x_D = 4 \implies x_D = 1$
$2 - y_D = -1 \implies y_D = 3$
$\implies D(1,3)$
$\overrightarrow{BD} = (1-3)\vec{i} + (3+2)\vec{j} = -2\vec{i} + 5\vec{j}$
$|\overrightarrow{BD}| = \sqrt{(-2)^2 + 5^2} = \sqrt{4+25} = \sqrt{29}$

b)

Šiljasti kut paralelograma je kut pri vrhu *A* (nacrtajte sliku)

Naći ćemo vektore $\overrightarrow{AB}$ i $\overrightarrow{AD}$ te njihove duljine.

$\overrightarrow{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} = 4\vec{i} - \vec{j}$, $\left|\overrightarrow{AB}\right| = \sqrt{4^2 + 1^2} = \sqrt{17}$

$\overrightarrow{AD} = (x_D - x_A)\vec{i} + (y_D - y_A)\vec{j} = 2\vec{i} + 4\vec{j}$, $\left|\overrightarrow{AD}\right| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$

$$\cos\alpha = \frac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{AD}\right|} = \frac{(4\vec{i} - \vec{j})(2\vec{i} + 4\vec{j})}{\sqrt{17}\cdot 2\sqrt{5}} = \frac{4\cdot 2 - 1\cdot 4}{2\sqrt{85}} = \frac{4}{2\sqrt{85}} = \frac{2}{\sqrt{85}}$$

$$\alpha = \arccos\frac{2}{\sqrt{85}}$$

$$\alpha = 77°28'$$

3.

Vektor $\overrightarrow{AD}$ prikaži kao linearnu kombinaciju vektora $\overrightarrow{AB}$ i $\overrightarrow{AC}$ ako je $A(-2,1)$, $B(-1,-1)$, $C(1,2)$ i $D(1,9)$.

$$\overrightarrow{AB} = (-1+2)\vec{\imath} + (-1-1)\vec{\jmath} = \vec{\imath} - 2\vec{\jmath}$$
$$\overrightarrow{AC} = (1+2)\vec{\imath} + (2-1)\vec{\jmath} = 3\vec{\imath} + \vec{\jmath}$$
$$\overrightarrow{AD} = (1+2)\vec{\imath} + (9-1)\vec{\jmath} = 3\vec{\imath} + 8\vec{\jmath}$$
$$\overrightarrow{AD} = \alpha\overrightarrow{AB} + \beta\overrightarrow{AC}$$
$$\alpha(\vec{\imath} - 2\vec{\jmath}) + \beta(3\vec{\imath} + \vec{\jmath}) = 3\vec{\imath} + 8\vec{\jmath}$$
$$(\alpha + 3\beta)\vec{\imath} + (-2\alpha + \beta)\vec{\jmath} = 3\vec{\imath} + 8\vec{\jmath}$$

$$\alpha + 3\beta = 3 / \cdot 2$$
$$-2\alpha + \beta = 8$$

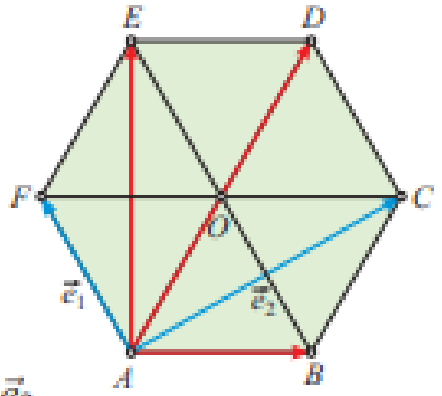
$$2\alpha + 6\beta = 6$$
$$-2\alpha + \beta = 8$$

$$7\beta = 14 \implies \beta = 2$$
$$\alpha + 6 = 3 \implies \alpha = -3$$
$$\overrightarrow{AD} = -3\overrightarrow{AB} + 2\overrightarrow{AC}$$

4.

Točke A, B, C, D, E i F vrhovi su pravilnog šesterokuta. Ako je $\overrightarrow{AF} = \vec{e}_1$, $\overrightarrow{AC} = \vec{e}_2$, prikaži vektore $\overrightarrow{AB}$, $\overrightarrow{AD}$ i $\overrightarrow{AE}$ kao linearnu kombinaciju vektora $\vec{e}_1$ i $\vec{e}_2$.

$$\overrightarrow{AB} = \vec{e}_2 + \overrightarrow{CB} = \vec{e}_2 + \overrightarrow{OA}$$
$$= \vec{e}_2 + \overrightarrow{OF} + \overrightarrow{FA} = \vec{e}_2 - \overrightarrow{AB} - \vec{e}_1$$
$$\implies 2\overrightarrow{AB} = \vec{e}_2 - \vec{e}_1 \implies \overrightarrow{AB} = \frac{1}{2}\vec{e}_2 - \frac{1}{2}\vec{e}_1;$$
$$\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD} = \vec{e}_1 + \overrightarrow{AF} = \vec{e}_1 + \vec{e}_2;$$
$$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \frac{1}{2}\vec{e}_2 - \frac{1}{2}\vec{e}_1 + 2\vec{e}_1 = \frac{3}{2}\vec{e}_1 + \frac{1}{2}\vec{e}_2.$$



5. Zadane su točke $B(1,-6), C(3,8)$

a) Odredi nepoznatu koordinatu vrha $A(x,-4)$ trokuta *ABC* tako da trokut bude pravokutan s pravim kutom u vrhu *A*.

b) Odredi vektor $\overrightarrow{b}$ kolinearan s vektorom $\overrightarrow{BC}$ ako je $|\vec{b}| = 5\sqrt{2}$

a) Kako je pravi kut u vrhu *A*, koristimo uvjet okomitosti $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$$\overrightarrow{AB} = (1-x)\vec{i} + (-6+4)\vec{j} = (1-x)\vec{i} - 2\vec{j}$$

$$\overrightarrow{AC} = (3-x)\vec{i} + (8+4)\vec{j} = (3-x)\vec{i} + 12\vec{j}$$

Sada uvrstimo u $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$$[(1-x)\vec{i} - 2\vec{j}] \cdot [(3-x)\vec{i} + 12\vec{j}] = 0$$
$$(1-x)(3-x) - 24 = 0$$
$$3 - x - 3x + x^2 - 24 = 0$$
$$x^2 - 4x - 21 = 0$$
$$x_1 = 7,\ x_2 = -3$$
$$A_1(7,-4),\ A_2(-3,-4)$$

b) Uvjet kolinearnosti: $\overrightarrow{b} = k \cdot \overrightarrow{BC},\ k \in R,\ \overrightarrow{b} = x\vec{i} + y\vec{j}$

$$\overrightarrow{BC} = (3-1)\vec{i} + (8+6)\vec{j} = 2\vec{i} + 14\vec{j}$$
$$x\vec{i} + y\vec{j} = k(2\vec{i} + 14\vec{j})$$
$$x\vec{i} + y\vec{j} = 2k\vec{i} + 14k\vec{j}$$
$$x = 2k,\ y = 14k$$

Znamo da je $|\vec{b}| = 5\sqrt{2}$ pa imamo:

$$|\vec{b}| = \sqrt{x^2 + y^2}$$
$$5\sqrt{2} = \sqrt{4k^2 + 196k^2}$$
$$5\sqrt{2} = \sqrt{200k^2}$$
$$5\sqrt{2} = 10\sqrt{2}|k| \ / : 10\sqrt{2}$$
$$|k| = \frac{1}{2},\ k_1 = \frac{1}{2},\ k_2 = -\frac{1}{2}$$

pa nađemo

$$x_1 = 2 \cdot \frac{1}{2} = 1$$
$$y_1 = 14 \cdot \frac{1}{2} = 7$$
$$x_2 = 2 \cdot \left(-\frac{1}{2}\right) = -1$$
$$y_2 = -7$$

i na kraju

$$\overrightarrow{b_1} = \vec{i} + 7\vec{j}$$
$$\overrightarrow{b_2} = -\vec{i} - 7\vec{j}$$